Chapter 2

Motion along a straight line

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In this chapter we will study kinematics, the description of the motion. i.e., how objects move along a straight line.

The following parameters will be defined:

Displacement الازاحه

Average velocity السرعه المتوسطه

Average speed السرعه العدديه المتوسطه

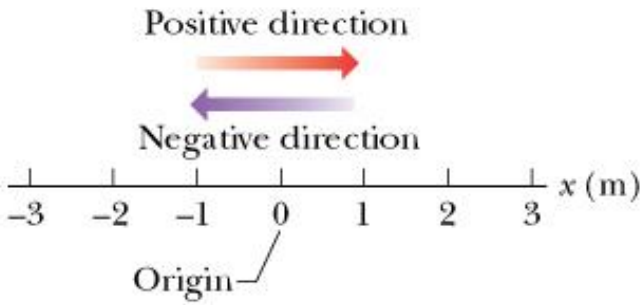
Instantaneous velocity السرعه اللحظيه

Average and instantaneous acceleration التسارع المتوسط و اللحظي

Motion

We will assume that the moving objects are "**particles,**" i.e., we restrict our discussion to the motion of objects for which all the points move in the same way.

The causes of the motion will not be investigated.



Consider an object moving along a straight line taken to be the *x*-axis. The object's position at any time *t* is described by its coordinate $x(t)$ defined with respect to the origin *O*. The coordinate *x* can be positive or negative depending whether the object is located on the positive or the negative part of the *x*-axis.

Displacement

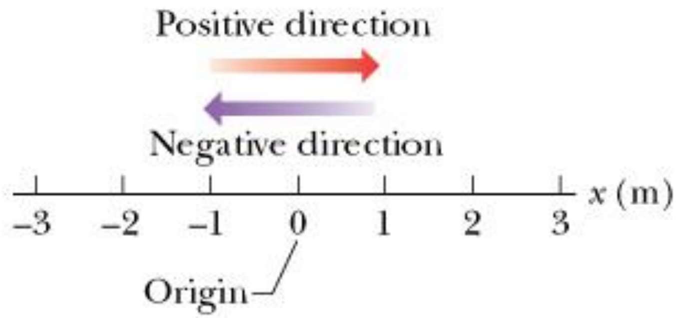
If an object moves from position x_1 to position x_2 , the change in position is described by the displacement :

$$\boxed{\Delta x = x_2 - x_1}$$

For example if $x_1 = 5$ m and $x_2 = 12$ m then $\Delta x = 12 - 5 = 7$ m. The positive sign of Δx indicates that the motion is along the positive x-direction.

If instead the object moves from $x_1 = 5$ m and $x_2 = 1$ m then $\Delta x = 1 - 5 = -4$ m. The negative sign of Δx indicates that the motion is along the negative x-direction.

Displacement is a vector quantity that has both magnitude and direction.



الازاحه تتطلب معرفه الموضع النهائي و الابتدائي بغض النظر عن عدد الامتار المقطوعه

Average Velocity

Average velocity, or v_{avg}, is defined as the displacement over the time duration.

$$\boxed{v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}}$$

Here x_2 and x_1 are the positions $x(t_2)$ and $x(t_1)$, respectively.

The **time interval** Δt is defined as $\Delta t = t_2 - t_1$.

Average velocity is a vector quantity that has both magnitude and direction.

The units of v_{avg} are m/s.

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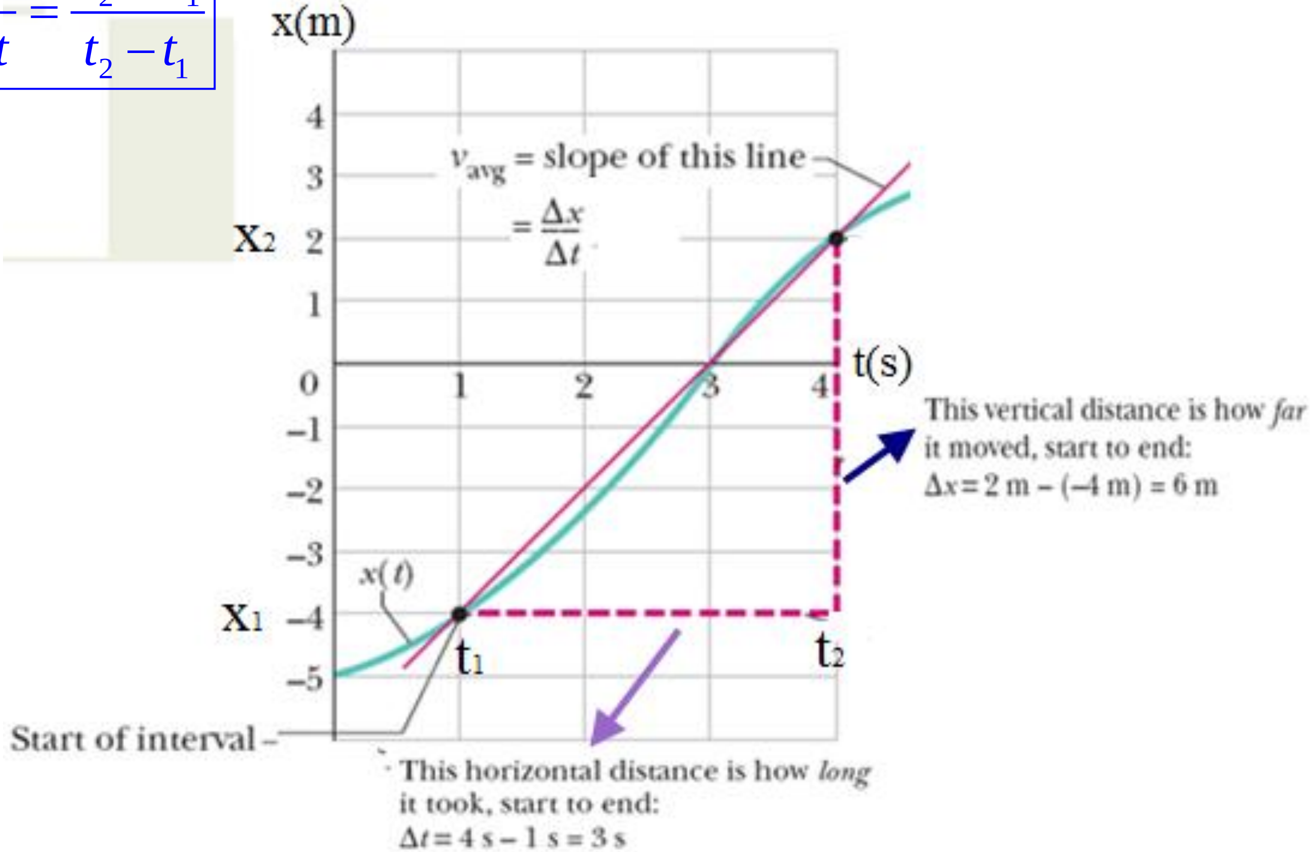
2.4 Average Velocity

The magnitude of the slope of the x-t graph gives the average velocity

$$\boxed{v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}}$$



Here, the average velocity is:

$$v_{\text{avg}} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{2-(-4)}{4-1} = \frac{6\text{ m}}{3\text{ s}} = 2\text{ m/s}$$

Average Speed

The average speed is defined in terms of the **total distance** traveled in a time interval Δt (and not the displacement Δx as in the case of v_{avg}).

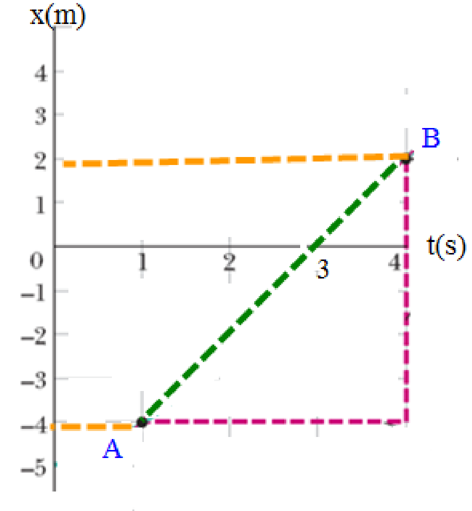
$$\boxed{s_{avg} = \frac{\text{total distance}}{\Delta t}}$$

The average speed does not include direction.

Instantaneous Velocity

In order to describe how fast an object moves at any time t we introduce the notion of instantaneous velocity v. Instantaneous velocity is defined as the limit of the average velocity determined for a time interval Δt as we let $\Delta t \rightarrow 0$.

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$



From its definition instantaneous velocity is the first derivative (التفاضل الأول) of the position coordinate x with respect to time.

Speed

We define speed as the magnitude of an object's velocity vector.

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Average Acceleration

We define the average acceleration a_{avg} between t_1 and t_2 as:

$$a_{avg} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

The SI units for acceleration are m/s^2

Instantaneous Acceleration

If we take the limit of a_{avg} as $\Delta t \to 0$ we get the instantaneous acceleration a, which describes how fast the velocity is changing at any time t.

$$a = \frac{dv}{dt}, \quad a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

Note: If a particle has *the same sign for velocity and acceleration*, then that particle is speeding up.

Conversely, if a particle has *opposite signs for the velocity and acceleration*, then the particle is slowing down.

Sample Problem 2-3:

The position of a particle moving on an x axis is given by

$$x = 7.8 + 9.2t - 2.1t^3, \qquad (2\text{-}5)$$

with x in meters and t in seconds. What is its velocity at $t = 3.5$ s? Is the velocity constant, or is it continuously changing?

KEY IDEA Velocity is the first derivative (with respect to time) of the position function $x(t)$.

Calculations: For simplicity, the units have been omitted from Eq. 2-5, but you can insert them if you like by changing the coefficients to 7.8 m, 9.2 m/s, and $-2.1\ \text{m/s}^3$. Taking the derivative of Eq. 2-5, we write

$$v = \frac{dx}{dt} = \frac{d}{dt}(7.8 + 9.2t - 2.1t^3),$$

which becomes

$$v = 0 + 9.2 - (3)(2.1)t^2 = 9.2 - 6.3t^2. \qquad (2\text{-}6)$$

At $t = 3.5$ s,

$$v = 9.2 - (6.3)(3.5)^2 = -68\ \text{m/s}. \quad \text{(Answer)}$$

At $t = 3.5$ s, the particle is moving in the negative direction of x (note the minus sign) with a speed of 68 m/s. Since the quantity t appears in Eq. 2-6, the velocity v depends on t and so is continuously changing.

ملخص ما تم دراسته:

Displacement: $$\boxed{\Delta x = x_2 - x_1}$$

Average Velocity: $$\boxed{v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}}$$

Average Speed: $$\boxed{s_{\text{avg}} = \frac{\text{total distance}}{\Delta t}}$$

Instantaneous Velocity: $$\boxed{v = \frac{dx}{dt}}$$

Average Acceleration: $$\boxed{a_{\text{avg}} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}}$$

Instantaneous Acceleration: $$\boxed{a = \frac{dv}{dt}, \quad a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2 x}{dt^2}}$$

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Sample Problem 2-4:

A particle's position on the x axis is given by

$$x = 4 - 27t + t^3,$$

with x in meters and t in seconds.

Because position x depends on time t, the particle must be moving. Find the particle's velocity function $v(t)$ and acceleration function $a(t)$.

The position: $\boxed{x = 4 - 27t + t^3}$

The velocity: $\boxed{v = \dfrac{dx}{dt}}$

$\boxed{v = -27 + 3t^2}$

The acceleration: $\boxed{a = \dfrac{dv}{dt}}$

$\boxed{a = 6t}$

2.7: Constant acceleration

When the acceleration is constant, its average an instantaneous values are the same.

$a = a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{v - v_0}{t - 0}$ **means that** $\boxed{v = v_0 + at.}$ **……(1)**

The velocity at t=0 is the initial velocity (v_o) and v is the final velocity .

Similarly, $v_{avg} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{x - x_0}{t - 0}$ which means tha $x = x_0 + v_{avg}t,$ where $v_{avg} = \frac{1}{2}(v_0 + v)$

finally leading to $\boxed{x - x_0 = v_0 t + \frac{1}{2}at^2.}$ **……(2)**

Eliminating t from the Equations (1) and (2): $t = \frac{v - v_0}{a}$

$$\boxed{v^2 = v_0^2 + 2a(x - x_0).}$$ **……(3)**

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Substituting a from equation (1) into equation (2):

$$v = v_0 + at. \quad (1)$$

$$x - x_0 = v_0 t + \tfrac{1}{2}at^2. \quad (2)$$

We get, $x - x_0 = \dfrac{1}{2}(v_0 + v)t$ (4)

Substituting v_o from equation (1) into equation (2):

Then, $x - x_0 = vt - \dfrac{1}{2}at^2$ (5)

TABLE 2-1

Equations for Motion with Constant Acceleration[a]

Equation Number	Equation	Missing Quantity
2-11	$v = v_0 + at$	$x - x_0$
2-15	$x - x_0 = v_0 t + \frac{1}{2}at^2$	v
2-16	$v^2 = v_0^2 + 2a(x - x_0)$	t
2-17	$x - x_0 = \frac{1}{2}(v_0 + v)t$	a
2-18	$x - x_0 = vt - \frac{1}{2}at^2$	v_0

CHECKPOINT 4 The following equations give the position $x(t)$ of a particle in four situations: (1) $x = 3t - 4$; (2) $x = -5t^3 + 4t^2 + 6$; (3) $x = 2/t^2 - 4/t$; (4) $x = 5t^2 - 3$. To which of these situations do the equations of Table 2-1 apply?

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Equations for Motion with Constant Acceleration[a]

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(1) $x = 3t - 4$

$$x = 3t - 4$$
$$v = 3$$
$$a = 0$$

The acceleration is constant.

(2) $x = -5t^3 + 4t^2$

$$x = -5t^3 + 4t^2$$
$$v = -15t^2 + 8t$$
$$a = -30t + 8$$

(3) $x = \dfrac{2}{t^2} - \dfrac{4}{t}$

$$x = \frac{2}{t^2} - \frac{4}{t}$$
$$x = 2t^{-2} - 4t^{-1}$$
$$v = -4t^{-3} + 4t^{-2}$$
$$a = 12t^{-4} - 8t^{-3}$$

(4) $x = 5t^2 - 3$

$$x = 5t^2 - 3$$
$$v = 10t$$
$$a = 10$$

The acceleration is constant.

Sample Problem 2-5:

The head of a woodpecker is moving forward at a speed of 7.49 m/s when the beak makes first contact with a tree limb. The beak stops after penetrating the limb by 1.87 mm. Assuming the acceleration to be constant, find the acceleration magnitude in terms of g.

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2-16	$v^2 = v_0^2 + 2a(x - x_0)$	t
2-17	$x - x_0 = \frac{1}{2}(v_0 + v)t$	a
2-18	$x - x_0 = vt - \frac{1}{2}at^2$	v_0

The initial velocity v_o =7.49 m/s^2

x-x_o=1.87 mm=1.87x 10^{-3} m

Because he stops, the final velocity=0

In this problem we do not know the time, so the only equation we can use is (2-16)

$$\boxed{v^2 = v_0^2 + 2a\left(x - x_o\right)}$$

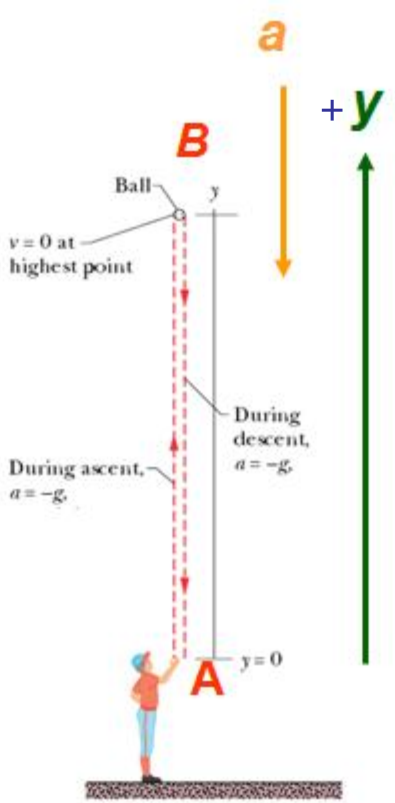
$$\boxed{0 = (7.49)^2 + 2a\left(1.87x10^{-3}\right)}$$

$$\boxed{a = -1.5x10^4 \ m/s^2}$$

$$\boxed{\frac{a}{g} = \frac{1.5x10^4}{9.8} = 1.53x10^3} \Rightarrow \boxed{a = 1.53x10^3 \ g}$$

Free Fall

Close to the surface of the Earth all objects move toward the center of the Earth with an acceleration whose magnitude is constant and equal to 9.8 m/s^2. We use the symbol *g* to indicate the acceleration of an object in free fall.



If we take the *y*-axis to point upward then the acceleration of an object in free fall ***a = -g*** and the equations for free fall take the form:

$$v = v_0 - gt \qquad (1)$$

$$y = y_o + v_0 t - \frac{gt^2}{2} \qquad (2)$$

$$v^2 - v_0^2 = -2g\left(y - y_o\right) \qquad (3)$$

$$y - y_o = \frac{1}{2}(v_0 + v)t \qquad (4)$$

$$y - y_o = vt - \frac{1}{2}at^2 \qquad (5)$$

Note:, the velocity can be positive (upward motion from point *A* to point *B*). It is momentarily zero at point *B*. The velocity becomes negative on the downward motion from point *B* to point *A*.

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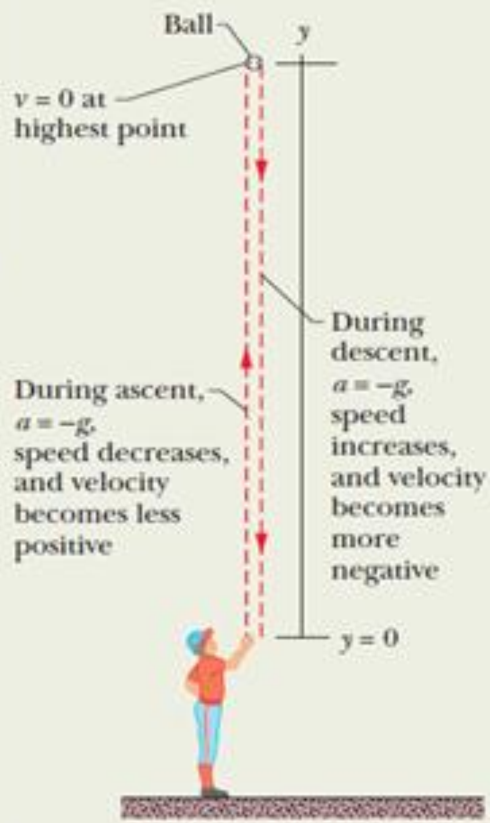
Sample problem 2-8

In Fig. 2-11, a pitcher tosses a baseball up along a y axis, with an initial speed of 12 m/s.

(a) How long does the ball take to reach its maximum height?



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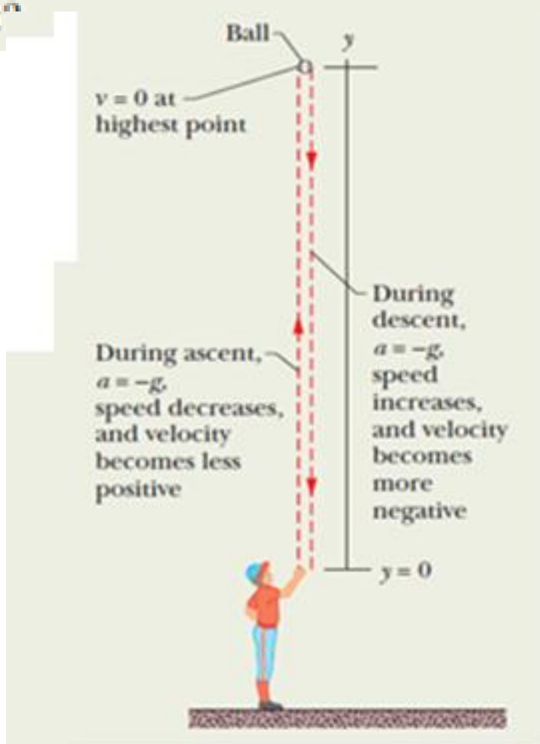
2-17	$x - x_0 = \frac{1}{2}(v_0 + v)t$	a
2-18	$x - x_0 = vt - \frac{1}{2}at^2$	v_0

The initial velocity =12 m/s.
At maximum point v=0.
The time at the highest point=?
Use (2-11):

$$V=V_o+at$$

$$t = \frac{v - v_0}{a} = \frac{0-12}{-9.8} = 1.2\ s$$

(b) What is the ball's maximum height above its release point?



$$v^2 = v_0^2 - 2g\left(y - y_o\right)$$

$$0 = v_0^2 - 2g\left(y\right)$$

$$0 = (12)^2 - 2(9.8)\left(y\right)$$

$$y = 7.3 \;\; m$$

TABLE 2-1

Equations for Motion with Constant Acceleration[a]

Equation Number	Equation	Missing Quantity
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2-16	$v^2 = v_0^2 + 2a(x - x_0)$	t
2-17	$x - x_0 = \frac{1}{2}(v_0 + v)t$	a
2-18	$x - x_0 = vt - \frac{1}{2}at^2$	v_0

(c) How long does the ball take to reach a point 5.0 m above its release point?

$$y - y_0 = v_0 t - \frac{1}{2}gt^2$$

$$y = v_0 t - \frac{1}{2}gt^2$$

$$5 = (12)t - \frac{1}{2}(9.8)t^2$$

$$4.9t^2 - 12t + 5 = 0$$

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = 0.53 \text{ s} \qquad \text{and} \quad t = 1.9 \text{ s}$$